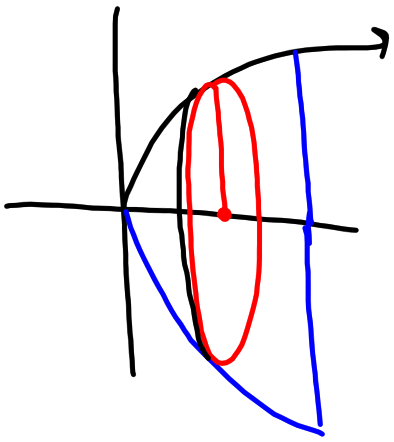
7.3 Volumes by Revolution

Ex 1) Revolve $y = \sqrt{x}$ around the x-axis, $0 \le x \le 5$. Find the volume of the solid.



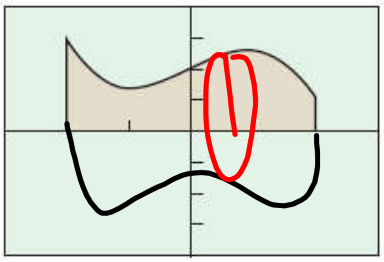
Disks/Washers

area of disk : πr^2

$= \pi(\sqrt{x})^2 = \pi x$

$$V = \int_0^5 \pi x \cdot dx = \frac{\pi x^2}{2}\Big|_0^5 = \frac{25\pi}{2}$$

The region between the graph of $f(x) = 2 + x\cos x$ and the x-axis over the interval $[-2, 2]$ is revolved about the x-axis to generate a solid. Find the volume of the solid.



[–3, 3] by [–4, 4]

Figure 7.18 The region in Example 2.

A of cross-section: $\pi(2+x\cos x)^2$

$$V = \int_{-2}^{2} \pi (2+x\cos x)^2 \, dx \approx 52.429$$

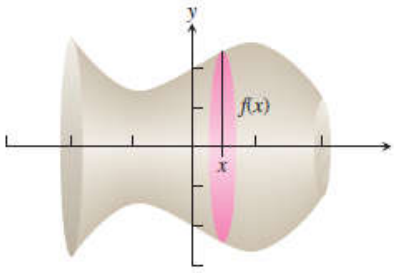
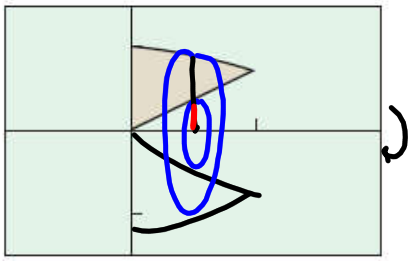


Figure 7.19 The region in Figure 7.18 is revolved about the x-axis to generate a solid. A typical cross section is circular, with radius $f(x) = 2 + x\cos x$. (Example 2)

EXAMPLE 3 Washer Cross Sections

The region in the first quadrant enclosed by the y-axis and the graphs of $y = \cos x$ and $y = \sin x$ is revolved about the x-axis to form a solid. Find its volume.



$[-\pi/4, \pi/2]$ by $[-1.5, 1.5]$

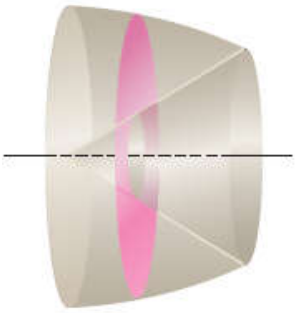


Figure 7.20 The region in Example 3.

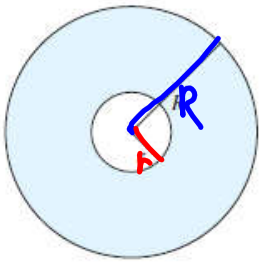


Figure 7.22 The area of a washer is $\pi R^2 - \pi r^2$. (Example 3)

A of washer: $\pi R^2 - \pi r^2$

$\pi(R^2 - r^2)$

$R = \cos x$

$r = \sin x$

$\neq \pi(R-r)^2$

$A = \pi(\cos^2 x - \sin^2 x)$

$$V = \int_0^{\frac{\pi}{4}} \pi(\cos^2 x - \sin^2 x)\,dx = \int_0^{\frac{\pi}{4}} \pi \cdot \cos 2x\,dx$$

$= \frac{\pi}{2}$

TRY: The region enclosed by the graphs of y = 2x, y = x, and x = 1 is revolved around the x-axis to form a solid. Find the volume.